Macroeconomic Dynamics, 2013, Page 1 of 24. Printed in the United States of America.
doi:10.1017/S1365100513000163

ECONOMIC GROWTH AND EVOLUTION: PARENTAL PREFERENCE FOR QUALITY AND QUANTITY OF OFFSPRING

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This paper presents a quantitative analysis of the model developed by Galor and Moav [Galor, Oded and Omer Moav (2002) Natural selection and the origin of economic growth. *Quarterly Journal of Economics* 117(4), 1133–1191] in which agents vary genetically in their preference for quality and quantity of children. The simulation produces a pattern of income and population growth that resembles the period of Malthusian stagnation before the Industrial Revolution and the take-off into a modern growth era. We also investigate the stability of the modern growth era as an absorbing state of the model under the introduction of a strongly quantity-preferring genotype. We show that, given the absence of a scale effect of population in the model, the economy can regress to a Malthusian state under this change in the initial distribution of genotypes.

Keywords: Evolution, Natural Selection, Growth, Education, Human Capital

1. INTRODUCTION

Over the past thirty years, there has been increasing scientific interest in using evolutionary theory to explain human economic behavior. Since the advocacy of this approach by Becker (1976) and Hirshleifer (1977), Darwinian (1859) thinking has been used to explain the evolution of human risk preference [Rubin and Paul (1979)], time preference [Hansson and Stuart (1990); Rogers (1994); Robson and Samuelson (2007); Robson and Szentes (2008)], and the shape of utility functions [Netzer (2009)].[1] More recently, evolutionary theory has been applied to the emergence of modern economic growth.

Galor and Moav (2002) developed a unified growth model in which natural selection favors traits that affect the economic environment. This model was the first to use frequency changes in heritable traits to explain the shift of human populations from Malthusian stagnation to modern economic growth. Galor and

For their comments, we thank Oded Galor, Omer Moav, Tim Kam, two anonymous referees, and participants in seminars with the University of Western Australia economics discipline, UWA Centre for Evolutionary Biology, ETH Zurich experimental ecology and theoretical biology groups, and the University of Zurich's behavioral ecology group. Address correspondence to: Jason Collins, Economics (M251), Business School, The University of Western Australia, 35 Stirling Highway, Crawley, Perth, Western Australia 6009, Australia; e-mail: jason@jasoncollins.org.

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Moav proposed a gene-encoded preference for quality or quantity of children, which is similar to *r*/*K* selection in behavioral ecology [Planka (1970)]. The quantity–quality trade-off has been hypothesized as an economic factor by, among others, Becker (1960) and Becker and Lewis (1974). Becker et al. (1990) considered the link between the quantity–quality trade-off and economic growth.[2]

In the Galor and Moav model, individuals who invest a lot in the education of their children have a fitness advantage in the early stages of economic development. Fitness, as the term is used in biology, is the proportional contribution of a genotype to the gene pool of the next generation. As technological progress depends on human capital and the returns to education increase with technological progress, this positive feedback ultimately results in an escape from Malthusian stagnation. Galor and Moav noted that natural selection might favor other growth promoting traits. For example, Galor and Michalopoulos (2012) suggested that entrepreneurial spirit creates a selective advantage in the early stages of economic development, whereas less entrepreneurially spirited individuals do well in mature economies. The positive feedback between entrepreneurial spirit and economic development lifts the economy out of Malthusian stagnation.

Galor and Moav investigated the dynamics of their model analytically using phase diagrams. In this paper, their model is analyzed numerically by simulation. The method is similar to the one that Lagerlof (2006) used to simulate the model of Galor and Weil (2000).[3] The advantage of simulation is that it allows exploration of more richly specified models for which there exists no closed-form solution. In particular, it will be possible to consider the addition of a strongly quantity-preferring genotype to the population and to demonstrate that, given the absence of a scale effect of population in the model, the economy can regress to a Malthusian state under this change in the initial distribution of genotypes.

2. BACKGROUND

Besides Galor and Moav (2002), several other authors have applied evolutionary theory in the analysis of economic growth and the transition from the Malthusian state to modern rates of growth. In their seminal paper on the evolution of preferences for saving and labor supply, Hansson and Stuart (1990) proposed that human preferences depend on the availability of resources. Harsh natural environments select for genotypes that have a stronger preference for saving, leading to an equilibrium with low population density and high per-capita capital. Selected traits include a preference for work and accumulation of physical capital. This might explain why humans left the Malthusian state first in regions with harsh winters.

Clark (2007) suggested that selection for certain heritable characteristics accounted for the Industrial Revolution. Although open as to whether these traits were transmitted genetically or culturally, he found higher reproductive success among wealthy males in England between 1250 and 1800.[4] He hypothesized that individuals with favored traits such as a propensity to hard work and saving

increased in frequency during this time. This change in population composition could then have provided the basis for the Industrial Revolution.

The increasing availability of population genetic data, such as those of Cavalli-Sforza et al. (1994), has led to more research. Spolaore and Wacziarg (2009) linked differences in economic development with the genetic distance between populations, which depends on the time elapsed since two populations shared a common ancestor. They proposed that genetic distance increases income differences because it may act as a barrier to the diffusion of technological development between populations. As genetic distance is based on neutral genes that are not subject to selection pressure, their hypothesis does not necessarily rely on any genetically determined difference in traits between populations, although genetic distance may serve as a proxy for vertically transmitted characteristics that affect the diffusion of development.

Recently, Ashraf and Galor (2013) proposed that the geographic distance of a population from Africa has affected the level of growth and development across regions. They found that populations with elevated or reduced genetic diversity experienced the lowest level of economic development in preindustrial times, and that this pattern has persisted following the Industrial Revolution.[5] The indigenous populations of the Americas have the lowest level of genetic diversity because of the founder effect, whereas Africans have the highest.[6] They suggested that the hump-shaped relationship between genetic diversity and economic development is due to a trade-off between the costs and benefits of genetic diversity. A high level of genetic diversity expands production possibilities through complementarities in knowledge production, but reduces the efficiency of the aggregate production process as lower levels of trust and coordination between dissimilar individuals reduce cooperation and create the potential for socioeconomic disruption. As for the measure of genetic distance used by Spolaore and Wacziarg (2009), the measure of genetic diversity used by Ashraf and Galor is based on non-protein-coding regions of the genome, and accordingly, their hypothesis does not rely on genetically determined differences in traits between populations.

3. THE GALOR AND MOAV MODEL

Galor and Moav (2002) developed an overlapping generations model, with each agent living for two periods (childhood and adulthood). In childhood, agents are passive and receive education. During adulthood, agents decide on how much time to dedicate to work or childrearing and they choose the number of children and their education. Reproduction is asexual by a single parent.

Production in the economy occurs with inputs of labor, H_t, and a limited resource, X, which may be called land. H_t measures the aggregate quantity of efficiency units of labor at time t. Aggregate output, Y_t, is given by a constant-returns-to-scale technology:

$$Y_t = H_t^{1-\alpha}(A_t X)^{\alpha}, \quad \alpha \in (0, 1). \tag{1}$$

The level of technology, A_t, is determined endogenously in the model. $1-\alpha$ is the elasticity of output with respect to labor input.

Assuming there are no property rights in land, the return to land is zero and the wage per efficiency unit of labor, w_t, is the output per unit of labor, x_t,

$$w_t = x_t^{\alpha}, \tag{2}$$

where $x_t = A_t X / H_t$.

The population consists of two genotypes ($i = a, b$) with different preferences between the quality and quantity of their children. Genotype a is a quality-preferring genotype, whereas genotype b has a relative preference for quantity. The utility function is

$$u_t^i = (1-\gamma)\ln c_t^i + \gamma\left(\ln n_t^i + \beta^i \ln h_{t+1}^i\right), \tag{3}$$

$$\gamma \in (0,1); \quad \beta^i \in (0,1]; \quad i \in a, b,$$

where c_t^i is the consumption of an individual with genotype i in period t, n_t^i is the number of children, and h_{t+1}^i is the level of human capital of each child. The parameter γ measures the relative weight of children in the utility function and the parameter β^i determines the weight that a genotype-i individual gives to the quality of children. Both parameters are inherited without change by the subsequent generations.

In adulthood, agents have one unit of time that they allocate between childrearing and participation in the labor market. Potential income, z_t^i, is the maximum income that could be earned if the agent's entire time endowment were devoted to labor force participation. Because the wage rate is expressed per efficiency unit of labor, potential income is

$$z_t^i = w_t h_t^i = x_t^{\alpha} h_t^i. \tag{4}$$

A parent incurs a base time cost, τ, for each child, with an additional time cost to educate the child to the level of education e_{t+1}^i. The total cost of raising a family with n children is $n_t^i(\tau + e_{t+1}^i)$ and the time left for working is $1 - n_t^i(\tau + e_{t+1}^i)$. Thus, the budget constraint faced in adulthood is

$$c_t^i \leq w_t h_t^i \left[1 - n_t^i\left(\tau + e_{t+1}^i\right)\right]. \tag{5}$$

Human capital, which determines an agent's efficiency units of labor during adulthood, is a function of education and the technological environment. Education increases human capital, whereas technological progress reduces the usefulness of existing human capital. The function for human capital and the conditions it

must satisfy are as follows:

$$\begin{aligned}
&h^i_{t+1} = h\left(e^i_{t+1}, g_{t+1}\right), && g_{t+1} \equiv \left(A_{t+1} - A_t\right)/A_t, \\
&h_e\left(e^i_{t+1}, g_{t+1}\right) > 0, && h_{ee}\left(e^i_{t+1}, g_{t+1}\right) < 0, \\
&h_g\left(e^i_{t+1}, g_{t+1}\right) < 0, && h_{gg}\left(e^i_{t+1}, g_{t+1}\right) > 0, \\
&h_{eg}\left(e^i_{t+1}, g_{t+1}\right) > 0, && h\left(0, 0\right) = 1, \\
&\lim_{g\to\infty} h\left(0, g_{t+1}\right) = 0.
\end{aligned} \tag{6}$$

Human capital increases at a diminishing rate with education (e^i_{t+1}) and is eroded at a decreasing rate by technological progress (g_{t+1}). Technological progress strengthens the effect of education on human capital. Human capital is normalized to one in the absence of education and technological progress.

Substituting equations (5) and (6) into equation (3), a genotype i parent of generation t faces the following optimization problem:

$$\begin{aligned}
\left\{n^i_t, e^i_{t+1}\right\} = \operatorname{argmax} \Big\{&(1-\gamma)\ln w_t h^i_t \left[1 - n^i_t\left(\tau + e^i_{t+1}\right)\right] \\
&+ \gamma\left[\ln n^i_t + \beta^i \ln h\left(e^i_{t+1}, g_{t+1}\right)\right]\Big\},
\end{aligned} \tag{7}$$

subject to income being enough to meet the subsistence level of consumption $\tilde{c}$:

$$\begin{gathered}
w_t h^i_t \left[1 - n^i_t\left(\tau + e^i_{t+1}\right)\right] \geq \tilde{c}, \\
\left(n^i_t, e^i_{t+1}\right) \geq 0.
\end{gathered} \tag{8}$$

The fertility of a genotype-i individual varies across three scenarios. These are where the subsistence constraint does not bind, where it binds, and where potential income is insufficient to meet the subsistence level of consumption. Taking the first-order condition of equation (7) with respect to n^i_t determines fertility when the constraint does not bind. Solving equation (8) as an equality gives fertility where the constraint binds. No children are born when the parent is reduced to the subsistence level of consumption. These three scenarios are shown in the equation

$$n^i_t = \begin{cases} \dfrac{\gamma}{\tau + e^i_{t+1}} & \textit{if } z^i_t \geq \tilde{z} \\[2ex] \dfrac{1 - \tilde{c}/z^i_t}{\tau + e^i_{t+1}} & \textit{if } \tilde{c} \leq z^i_t \leq \tilde{z} \\[2ex] 0 & \textit{if } z^i_t \leq \tilde{c}, \end{cases} \tag{9}$$

where $\tilde{z} \equiv \tilde{c}/(1-\gamma)$.

Equation (9) indicates that the number of children depends positively on potential income and negatively on the time cost of childrearing. Above the critical

value $\tilde{z}$, only the time cost of childrearing matters. No children are born when the parent is reduced to the subsistence level of consumption.

Taking the first-order condition of equation (7) with respect to the second choice variable e_{t+1} gives

$$\beta^i h_e\left(e^i_{t+1}, g_{t+1}\right) - \frac{h\left(e^i_{t+1}, g_{t+1}\right)}{\left(\tau + e^i_{t+1}\right)} \begin{cases} = 0 & \mathit{if}\ e > 0 \\ \leq 0 & \mathit{if}\ e = 0 \end{cases}. \tag{10}$$

The first term represents the utility benefit of a marginal increase in investment in the quality of children. The utility benefit of education depends positively on the partial derivative of the human capital function h_e and the weight given to the quality of children in the utility function β^i. The second term is the utility benefit of a marginal increase in investment in the quantity of children. Optimal behavior requires that the marginal benefit of education equal the marginal benefit of additional children if the parent chooses a positive level of education.

The following condition ensures that the level of education is positive for those with the highest valuation for quality ($\beta^i = 1$) when technological progress is zero:

$$h_e(0, 0) > \frac{1}{\tau}. \tag{11}$$

If equation (11) is not satisfied, no agents will educate their children, leading to a permanent Malthusian state.

The average level of education in the population, e_t, is

$$\begin{aligned} e_t &= q_t e^a_t + (1 - q_t)e^b_t, \\ q_t &= \frac{L^a_t}{L^a_t + L^b_t} = \frac{L^a_t}{L_t}. \end{aligned} \tag{12}$$

q_t indicates the proportion of genotype a in the population, with L^a_t and L^b_t the numbers of genotype a and b individuals and L_t the total population. It is assumed that the rate of technological progress, g_{t+1}, which determines economic growth, is an increasing and concave function of the average level of education:

$$\begin{gathered} g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = \varphi(e_t), \\ \varphi_e > 0; \quad \varphi_{ee} < 0; \quad \varphi(0) = 0. \end{gathered} \tag{13}$$

Finally, the number of efficiency units of labor supplied by the population is

$$H_t = L^a_t f^a_t h^a_t + L^b_t f^b_t h^b_t = L_t\left[q_t f^a_t h^a_t + (1 - q_t) f^b_t h^b_t\right], \tag{14}$$

where f^i_t is the fraction of time used by genotype i for labor:

$$f^i_t = \begin{cases} 1 - \gamma & \mathit{if}\ z^i_t \geq \tilde{z} \\ \dfrac{\tilde{c}}{z^i_t} & \mathit{if}\ \tilde{c} \leq z^i_t \leq \tilde{z} \end{cases}. \tag{15}$$

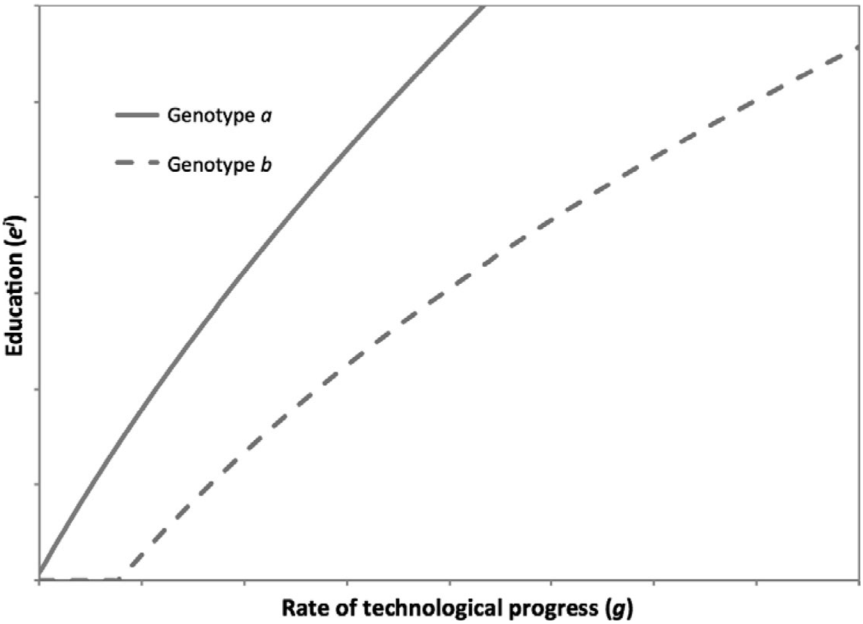

FIGURE 1. Educational response curves.

Equation (15) reflects the growing allocation of time to child rearing when potential income increases. When income reaches the critical value $\tilde{z}$, the fraction of time used for child rearing reaches a maximum of γ, leaving the fraction $1-\gamma$ for labor.

Using equations (14) and (15), the aggregate labor supply is

$$H_t = \begin{cases} L_t(1-\gamma)\left[q_t h_t^a + (1-q_t)h_t^b\right] & \text{if } z_t^a \geq \tilde{z} \text{ and } z_t^b \geq \tilde{z} \\ L_t\left[q_t(1-\gamma)h_t^a + (1-q_t)\left(\tilde{c}/z_t^b\right)h_t^b\right] & \text{if } z_t^a \geq \tilde{z} \text{ and } \tilde{c} \leq z_t^b \leq \tilde{z} \\ L_t\left[q_t\left(\tilde{c}/z_t^a\right)h_t^a + (1-q_t)\left(\tilde{c}/z_t^b\right)h_t^b\right] & \text{if } \tilde{c} \leq z_t^a \leq \tilde{z} \text{ and } \tilde{c} \leq z_t^b \leq \tilde{z} \end{cases}$$
$$\equiv H\left(L_t^a, L_t^b, e_t^a, e_t^b, g_t, z_t^a, z_t^b\right). \quad \textbf{(16)}$$

4. RESPONSE CURVES

Despite each genotype having a fixed preference for quality, this does not result in a fixed level of investment in education over time, as the return to education changes with the rate of technological progress. However, the curve for agents' educational response to the rate of technological progress is fixed. Figure 1 shows how much time each genotype invests into education at a given rate of technological progress, with the quality-preferring genotype a investing more in education at all rates of technological progress. The shape of the response curves is based on simulations

of the model in Section 8. The inequality (11) guarantees that quality-preferring genotype a parents always choose a positive level of education for their children. It is also possible to derive the slope of the educational response curve by applying the implicit function rule to equation (10).[7]

In the Galor and Moav model, the fitness of a genotype depends solely on the number of offspring, which determines its prevalence in the population over time. Which genotype has more children in turn depends on the rate of technological progress and economic growth. In the Malthusian state, the quality-preferring genotype has more children because education increases human capital and potential income. The rise in the prevalence of the quality-preferring genotype underpins slow technological progress in the Malthusian state. The rate of technological progress gradually increases until a threshold is reached at which it becomes worthwhile for the quantity-preferring genotype to invest in education. This threshold is given by point A in Figure 1. This positive feedback leads to an acceleration of technological progress and economic growth, putting an end to Malthusian stagnation.

When potential income exceeds the critical value $\tilde{z}$ for both genotypes, they both will invest the same proportion of time in raising children. However, because $h_g < 0$, technological progress degrades human capital, which makes it costly for the quality-preferring genotype to maintain the high level of human capital of its children. For this reason, the quantity-preferring genotype gains a fitness advantage during the period of economic growth that follows the Malthusian state.

Figure 2 shows the number of children of each genotype as a function of output per efficiency unit of labor, x_t, which determines the wage per unit of human capital. The fertility response curves are based on the optimum conditions in equation (9) and the definition of potential income in equation (4). The quality-preferring genotype can procreate at a lower level of x_t than the quantity-preferring genotype. Figure 2 illustrates the reversal in relative fitness of genotypes that occurs during economic development, at point B.

The higher fitness of the quality-preferring genotype in the Malthusian state and the quantity-preferring type in the modern growth state is akin to the classical r/K selection theory in evolutionary biology. Individuals that use the r strategy produce many offspring, each of which has a low probability of surviving to adulthood, whereas K strategists produce fewer offspring in which they invest more heavily, giving them a higher probability of surviving to adulthood.[8] r strategists exploit less crowded ecological niches, whereas K strategists are favored in more crowded environments. This behavior occurs in the Galor and Moav model. In the Malthusian state, where resources are scarce and the economy is effectively crowded, the quality-preferring genotype has higher fitness. In the modern growth regime, the economy has become uncrowded, giving higher fitness to the quantity-preferring genotype.

The educational and fertility response functions in Figures 1 and 2 allow each genotype to vary the level of education of its children and their number in response to technological progress. However, genotypes do not have this flexibility in other

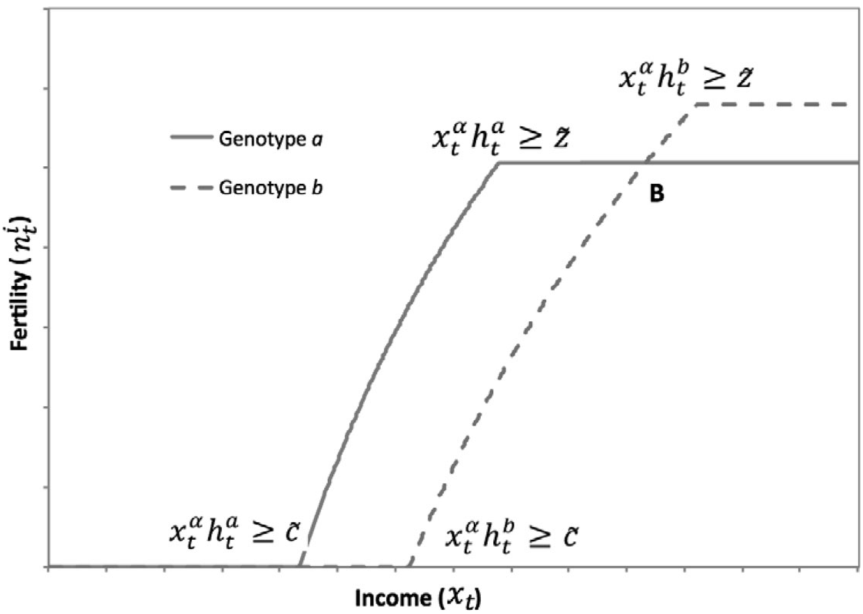

FIGURE 2. Fertility response curves.

dimensions. In particular, neither genotype fine-tunes its response to economic growth to optimize fitness. In the modern growth era, quality-preferring parents engage in a self-defeating strategy of overeducating their children. Additional flexibility in the educational response could materially affect model predictions.

5. FUNCTIONAL FORMS

To simulate the model, functional forms for h_{t+1}^i and g_{t+1} are needed. The following function for h_{t+1}^i matches most of the requirements given in equation (6):

$$h_{t+1}^i = h\left(e_{t+1}^i, g_{t+1}\right) = \frac{me_{t+1} + a}{e_{t+1} + rg_{t+1} + a}. \qquad \textbf{(17)}$$

This function does not fulfill the condition that $h_{eg} > 0$ for all values of e_{t+1}^i and g_{t+1}, but this is only a sufficient and not a necessary condition. Simulating the model of Galor and Weil, Lagerlof (2006) uses a similar functional form, with $m = r = 1$. Defining $a = \rho\tau$, $\rho \in (0,1)$, Lagerlof interpreted the parameter ρ as the portion of fixed time cost of childrearing that contributes towards the development of the base level of human capital.

The parameter m is included in equation (17) to allow the condition in equation (11), which ensures education by the quality-preferring genotype when there is zero economic growth, to be met. Using Lagerlof's definition of $a = \rho\tau$, m must

be greater than 1:

$$m > \frac{a}{\tau} + 1 = \rho + 1. \qquad \textbf{(18)}$$

The parameter r is selected to produce modern rates of education and economic growth.[9]

A simple functional form for equation (13) is the power function

$$g_{t+1} = ke_t^d; \quad 0 < d < 1; \quad k > 0. \qquad \textbf{(19)}$$

Using equations (10) and (17), the level of education that each genotype gives to its children is

$$\begin{aligned} e^i_{(t+1)} &= \max\left\{0,\ \frac{1}{2m}\left[(m(B^i - 1)(rg_{t+1} + a) - a(B^i + 1))\right.\right. \\ &\quad \left.\left. + \sqrt{\left(m(B^i - 1)(rg_{t+1} + a) - a(B^i + 1)\right)^2 + 4m((mB^i\tau - a)(rg_{t+1} + a) - aB^i\tau)}\right]\right\} \\ &\equiv \varphi^i\left(e_t\right). \end{aligned} \qquad \textbf{(20)}$$

This equation indicates that education in period $t + 1$ is a function of the rate of technological progress in period $t + 1$, which in turn is a function of the average level of education in period t. This link between education in one period and the next is crucial for the transition out of the Malthusian state.

6. THE DYNAMICAL SYSTEM

The dynamics of the system can be captured in a system of six difference equations that describe the behavior of the endogenous variables A_t, g_t, e_t^a, e_t^b, L_t^a and L_t^b. Before these equations are defined, z_t^i and n_t^i must be expressed in terms of the endogenous variables.

Collecting equations (2), (4), and (17) and given $H_t \equiv H(L_t^a, L_t^b, e_t^a, e_t^b, g_t, z_t^a, z_t^b)$, potential income per worker equals

$$z_t^i = w_t h_t^i = \left(\frac{A_t X}{H_t}\right)^\alpha \left(\frac{me_t^i + a}{e_t^i + rg_t + a}\right) \equiv z\left(A_t, L_t^a, L_t^b, e_t^a, e_t^b, g_t\right). \qquad \textbf{(21)}$$

Equations (9), (20), and (21) yield the number of children:

$$n_t^i = \left\{\begin{array}{ll} \dfrac{\gamma}{\left[\tau + \varphi^i(e_t)\right]} & if\ z_t^i \geq \tilde{z} \\ \dfrac{1 - \tilde{c}/z_t^i}{\left[\tau + \varphi^i(e_t)\right]} & if\ \tilde{c} \leq z_t^i \leq \tilde{z} \\ 0 & if\ z_t^i \leq \tilde{c} \end{array}\right\} \equiv \eta\left(A_t, L_t^a, L_t^b, e_t^a, e_t^b, g_t\right). \qquad \textbf{(22)}$$

TABLE 1. Parameter values

		Description	Value
Parameters	$1-\alpha$	Output elasticity of labor	0.6
	γ	Weight on children in utility function	0.259
	τ	Fixed time cost of children	0.20
	β^a	Preference for quality of genotype *a*	1
	β^b	Preference for quality of genotype *b*	0.9
	m	Weighting of education in production of human capital	2
	a	Portion of fixed cost time of raising child towards human capital	0.99τ
	X	Land	1
	$\tilde{c}$	Subsistence consumption constraint	1
	k	Growth function parameter	8.88
	d	Growth function parameter	0.5
	r	Responsiveness of human capital to economic growth	0.108
Initial values	L_0^a	Initial population of genotype *a*	0.007
	L_0^b	Initial population of genotype *b*	0.7
	e_0^a	Initial education of genotype *a*	0
	e_0^b	Initial education of genotype *b*	0
	A_0	Initial technology	1
	g_0	Initial rate of technological progress	0

The dynamical system for the six endogenous variables is

$$\begin{aligned}
A_{t+1} &= [1 + g(e_t)]\, A_t, \\
g_{t+1} &= g(e_t), \\
e_{t+1}^i &= \varphi^i(e_t); \quad i \in a, b, \\
L_{t+1}^i &= \eta\left(A_t, L_t^a, L_t^b, e_t^a, e_t^b, g_t\right) L_t^i; \quad i \in a, b.
\end{aligned}$$

7. PARAMETER VALUES

Table 1 lists the numerical values given to each parameter for the base case model. The preference parameter of the quality-preferring genotype, β^a, is set equal to 1. The preference parameter of the quantity-preferring type, β^b, must be high enough to allow for an exit from the Malthusian state. For any value of β^b below 0.894, the economy remains in the Malthusian state because technological progress never reaches a level high enough to induce the quantity-preferring genotype to invest in education. $\beta^b = 0.9$ is chosen because it produces a realistic level of education in the modern growth era.

The output elasticity of labor, 1–α, equals the labor share in national income if input factors are paid their marginal products. Clark (2010) provides estimates of the share in income from the Middle Ages to modern times. The labor share

increased from a low of 0.478 in the thirteenth century to above 0.6 in the early nineteenth century and to over 0.75 in the late twentieth century. We use a midpoint value of 0.6.

Population perturbations in the Malthusian state and during the transition period limit the potential values for the fixed time cost of children, τ. We can determine the range of τ for which population perturbations can be minimized by considering a population comprising solely genotype b. If $\tilde{c} \leq z_t^b \leq \tilde{z}$, as would be the case during the Malthusian era, and setting $\tilde{c}$, A_t, and X equal to 1, the population equation simplifies to the following first-order difference equation:

$$L_{t+1} = \frac{1}{\tau} L_t \left(1 - L_t^{\alpha}\right). \tag{23}$$

Setting $L_t = L_{t+1}$, the equilibrium for the population is $L^* = (1 - \tau)^{1/\alpha}$. Taking the derivative of equation (23) and substituting the equilibrium condition yields

$$\frac{\partial L_{t+1}}{\partial L_t} = \frac{1}{\tau} - \frac{1 + \alpha}{\tau}(1 - \tau). \tag{24}$$

The population equilibrium is unstable if $\partial L_{t+1}/\partial L_t > |1|$, with stability depending on the value of the fixed time cost of child rearing. For $\alpha = 0.4$, the equilibrium is unstable if $\tau \leq 0.1666$. Thus, for any value of the fixed time cost of children below this value, we can expect significant population perturbations. Subsequent testing demonstrated that even for values of τ slightly above that threshold, the presence of genotype a in the population results in ongoing perturbations. Setting $\tau = 0.20$ prevents extinctions and maintains reasonable population dynamics.[10]

An estimate for education expenditure, e, in the high-growth regime can be derived from OECD statistics. In 2009, education expenditures averaged 5.8% of GDP across OECD countries [OECD (2009, Table B.2.4)]. The model is calibrated to obtain an education level of genotype b individuals of 0.059, which is the OECD estimate for the United Kingdom. Given the predominance of genotype b, this is also the population average education in the modern growth regime.

The parameter γ, which is the same for both genotypes, determines the relative weight of children in the utility function. As modern fertility in developed countries is generally below replacement, γ is set to achieve zero population growth in the high-growth era; i.e., each parent has a single child. Setting $\gamma = 0.259$ and using the earlier values for τ and e yields $n = \gamma/(\tau + e) = 0.259/(0.20 + 0.059) = 1$.

Income per worker grew 2.3% per year in the United Kingdom from 1950 to 2008 [average annual growth 1960–2008 in Clark (2010, Table 33)]. Assuming 20 years per generation and using continuous compounding, the rate of technological progress g equals 216% per generation in the modern growth era.[11] With this g value and letting $d = 0.5$, the parameter k equals 8.88 in the growth equation (19).

The parameters a and m enter equation (17), which determines human capital h_t^i. Population perturbations increase if a is much less than 1 and m far above the level required for the quality-preferring type to educate their children when

there is no economic growth. We set $a = 0.99\tau$ and $m = 2$, satisfying inequality (18) and minimizing perturbations. The selection of $r = 0.108$ yields the chosen equilibrium values of education and economic growth in the modern growth era.

Finally, initial values must be chosen. The initial education is zero and hence the initial economic growth is zero. Initial technology, A, and land, X, are set equal to 1. At time zero, the numbers of genotype a and b individuals are assumed to be $L_0^a = 0.007$ and $L_0^b = 0.7$, with genotype a composing 1% of the population. This is close to the equilibrium population in the first period. Using equation (21), the level of income in the first period is approximately 1.25, which is above subsistence but such that the subsistence constraint still binds [as it is below $\tilde{z} = 1/(1-\gamma) = 1.35$].

8. SIMULATION RESULTS

The model explains the transition from Malthusian stagnation to modern economic growth, which occurred during the Industrial Revolution in the late 18th and early 19th centuries. When the simulation is initiated shortly before the beginning of the second millennium, the take-off occurs after about 45 generations or 900 years. The length of time to the take-off depends on the initial proportion of genotype a and b individuals. The transition phase from Malthusian stagnation to modern growth lasts about six generations or 120 years. During the transition phase, the rate of technological progress surges from less than 1% per annum to 5.7% and income growth rises to the modern growth rate of 2.3% per annum. Population growth increases until the time of the take-off and then reverses, dropping to zero during the transition phase. Figure 3 displays the behavior of the annual growth rates of technology, income, and population, and Figure 4 shows the log-levels of these variables. For population growth, income growth, and fertility, we present the results as five-generation moving averages, as short-run population perturbations require smoothing of the graphs to show the important trends and relative values in an effective visual manner.

A sudden increase in the education of genotype b individuals prompts the take-off in economic growth that leads out of the Malthusian state. As genotype b forms the majority of the population at all times, the average level of education in the population, which determines technological progress in equation (13), is approximately that of genotype b. Figure 5 shows the proportion of income spent on education by each genotype.

Figures 6 and 7, which relate to the fertility rate of each genotype and the genetic composition of the population, convey the fitness of the competing genotypes. Genotype a has a fitness advantage through the Malthusian era. The increase in the prevalence of genotype a fosters slow technological progress during the Malthusian era until a threshold is reached that makes it worthwhile for genotype b to invest in education.

After the economic take-off, genotype a parents begin to overinvest in education of their children to an extent that hampers their fitness. For this reason, the fitness

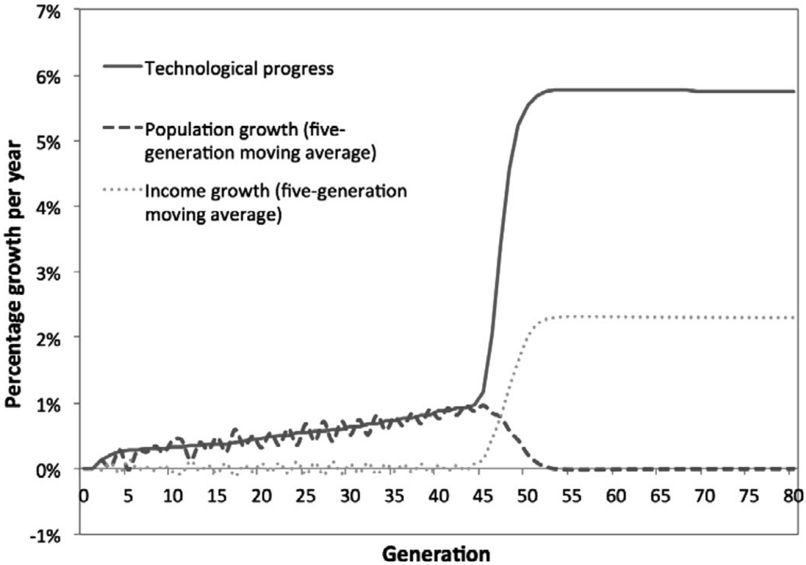

FIGURE 3. Annual growth rates of technology, population, and income.

of genotype *b* is higher than that of genotype *a* from the beginning of the transition and the prevalence of genotype *a* starts to decrease. Genotype *a*'s prevalence peaks at under 5% during the transition period. Thus, although the interaction between

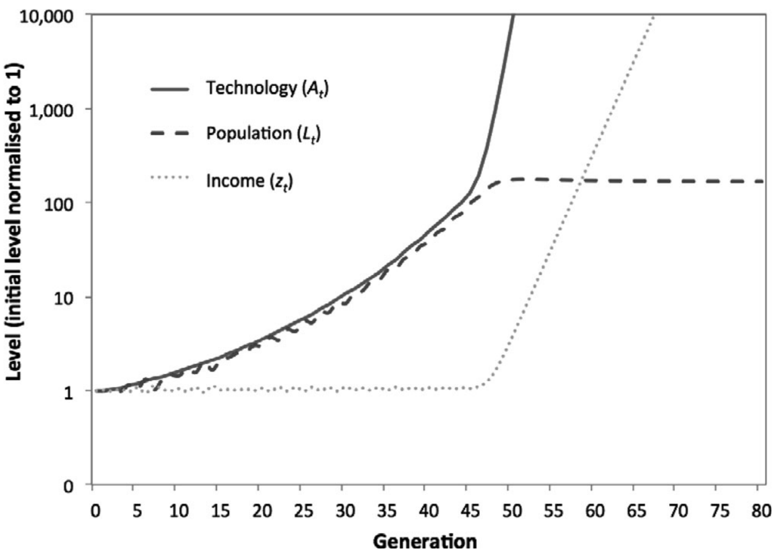

FIGURE 4. Level of technology, population, and income.

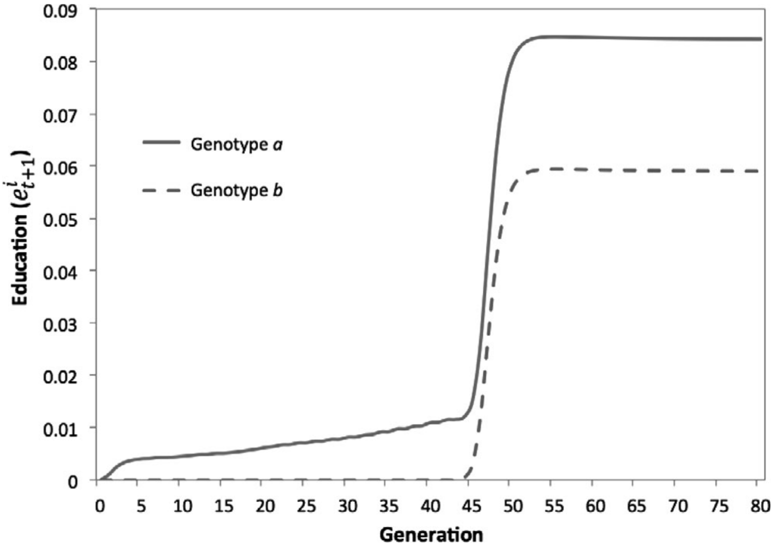

FIGURE 5. Education.

the two genotypes is crucial for the growth dynamics of the economy, a preference for quality always remains a rare trait.

During the Malthusian era, quality-preferring parents have both more and better educated children, whereas quantity-preferring parents have fewer and less educated children. Therefore, no quantity–quality trade-off is apparent *at the population level*. Although each individual makes a trade-off between quality and

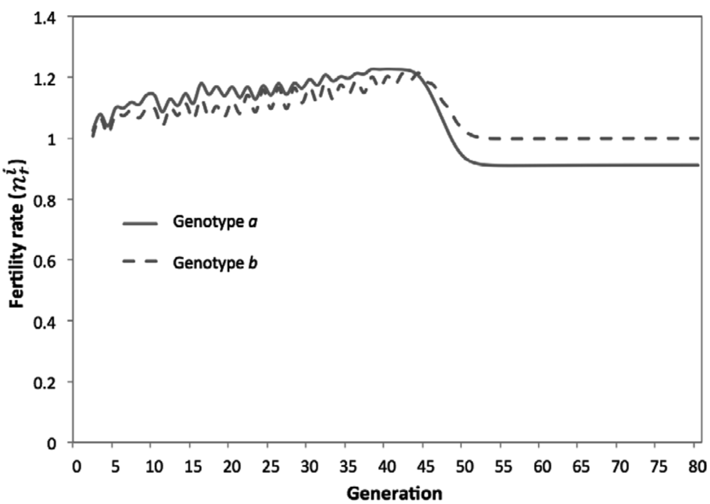

FIGURE 6. Fertility rate (five-generation moving average).

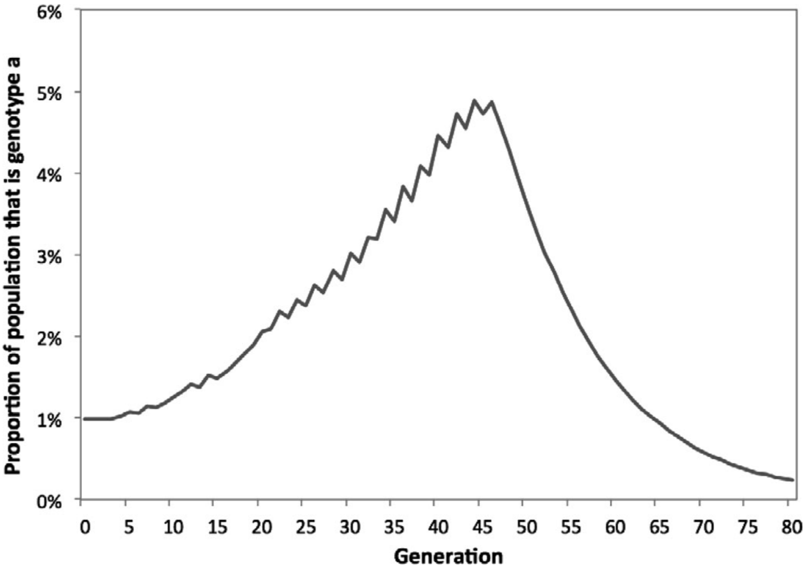

FIGURE 7. Proportion of population that is genotype *a*.

quantity, individuals who invest more in education also have higher fertility because of their own higher quality and income. From the beginning of the transition out of the Malthusian state, genotype *a* parents have better educated children, but fewer of them, whereas genotype *b* parents have more children with less education. From this point, the quantity–quality trade-off can be observed at the population level.

One empirical issue with the simulation concerns the timing of the demographic revolution. The simulated population stabilizes during the transition phase, whereas high rates of population growth persisted in industrial countries to the end of the 19th century. No set of parameters was found that would delay the demographic revolution in the simulation. One interpretation of the model that may reconcile the timing of the demographic revolution would be to consider the time cost of children and their education as part of the national income. In that case, income is already rising with fertility before the simulated transition out of Malthusian stagnation.[12]

9. SENSITIVITY OF THE MODERN GROWTH REGIME TO THE INTRODUCTION OF A STRONGLY QUANTITY-PREFERRING GENOTYPE

In the preceding simulation exercise, there are only two genotypes, with preferences for quality and quantity of children calibrated to achieve a transition of population, technology, and income that reflects the Industrial Revolution. In this section, we show how the presence of a strongly quantity-preferring genotype affects the model dynamics. The main finding is that, in the absence of a scale

effect in the model, a growing economy may regress to Malthusian conditions if a genotype that values education less than the other two genotypes is present in the population at the beginning of the simulated time period or is introduced exogenously during the simulation period. This finding is informative of the potential implications of migration (a new allele entering the population of interest from another population) or mutation (a spontaneous change in genotype).

Using the same functional forms as before, we simulate the model with three genotypes: the two genotypes a and b from the first simulation, plus a third strongly quantity-preferring genotype c. The quantity–quality preference parameters for the three genotypes are $\beta^a = 1.0$, $\beta^b = 0.9$, and $\beta^c = 0.75$. All other parameters of the model are the same as in Table 1, except the initial levels of the subpopulations, which are $L_0^a = 0.007$, $L_0^b = 0.7$, and $L_0^c = 0.007$. Thus, both the quality-preferring genotype a and the new strongly quantity-preferring genotype c are around 1% of the population at the beginning of the simulation.

Modifying equation (16), the aggregate labor supply is now

$$H_t = \begin{cases} L_t(1-\gamma)\left(q_t^a h_t^a + q_t^b h_t^b + q_t^c h_t^c\right) \\ \quad \textit{if } z_t^a \geq \tilde{z}, z_t^b \geq \tilde{z} \text{ and } z_t^c \geq \tilde{z} \\ L_t\left[(1-\gamma)\left(q_t^a h_t^a + q_t^b h_t^b\right) + q_t^c\left(\tilde{c}/z_t^c\right) h_t^c\right] \\ \quad \textit{if } z_t^a \geq \tilde{z},\ z_t^b \geq \tilde{z} \text{ and } \tilde{c} \leq z_t^c \leq \tilde{z} \\ L_t\left[(1-\gamma) q_t^a h_t^a + q_t^b\left(\tilde{c}/z_t^b\right) h_t^b + q_t^c\left(\tilde{c}/z_t^c\right) h_t^c\right] \\ \quad \textit{if } z_t^a \geq \tilde{z},\ \tilde{c} \leq z_t^b \leq \tilde{z} \text{ and } \tilde{c} \leq z_t^c \leq \tilde{z} \\ L_t\left[q_t^a(\tilde{c}/z_t^a) h_t^a + q_t^b\left(\tilde{c}/z_t^b\right) h_t^b + q_t^c\left(\tilde{c}/z_t^c\right) h_t^c\right] \\ \quad \textit{if } \tilde{c} \leq z_t^a \leq \tilde{z},\ \tilde{c} \leq z_t^b \leq \tilde{z} \text{ and } \tilde{c} \leq z_t^c \leq \tilde{z} \end{cases}$$

$$\equiv H\left(L_t^a,\ L_t^b,\ e_t^a,\ e_t^b,\ e_t^c,\ g_t,\ z_t^a,\ z_t^b,\ z_t^c\right). \tag{25}$$

q_t^a, q_t^b, and q_t^c indicate the proportions of genotypes a, b, and c in the population, respectively.

Figures 8 and 9, which display the growth rates and log levels of technology, population, and income, show that the first 60 generations of the simulation of the extended model including the strongly quantity-preferring genotype are similar to the baseline simulation in Section 8. The transition out of the Malthusian state occurs quite quickly, within six generations after generation 45. Population growth again peaks early during the transition phase, and the population then stabilizes. However, the subsequent growth era lasts for only about 20 generations, or 400 years. Economic growth abates because there is a renewed increase in population and a decline in the average level of investment in education after about generation 60, which does not occur in the model with only two genotypes. By generation 95, technological progress has ended and income growth is negative. The fall in

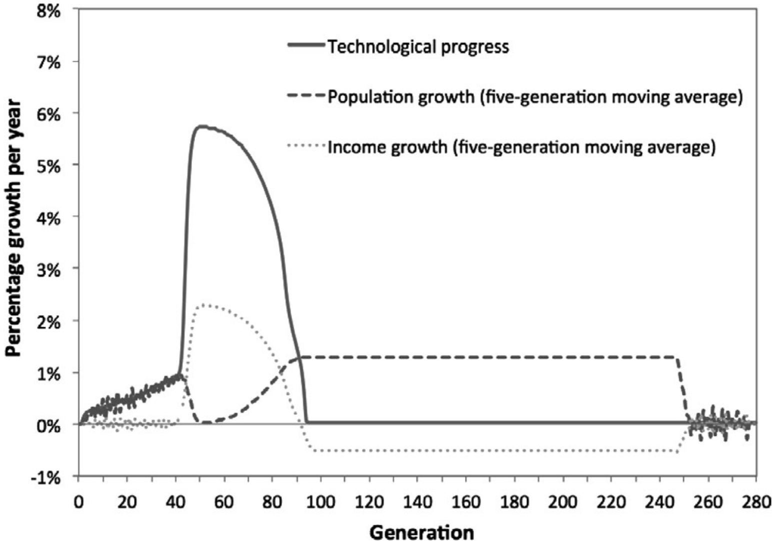

FIGURE 8. Annual growth rate of technology, population, and income.

per capita income continues until it has returned to the initial Malthusian level. Because technological progress is permanent, the economy supports a higher population during the second Malthusian era.

Figures 10, 11, and 12 reveal the behavior of education, fertility, and the genetic makeup of the population. In the initial Malthusian state, genotype a has the highest

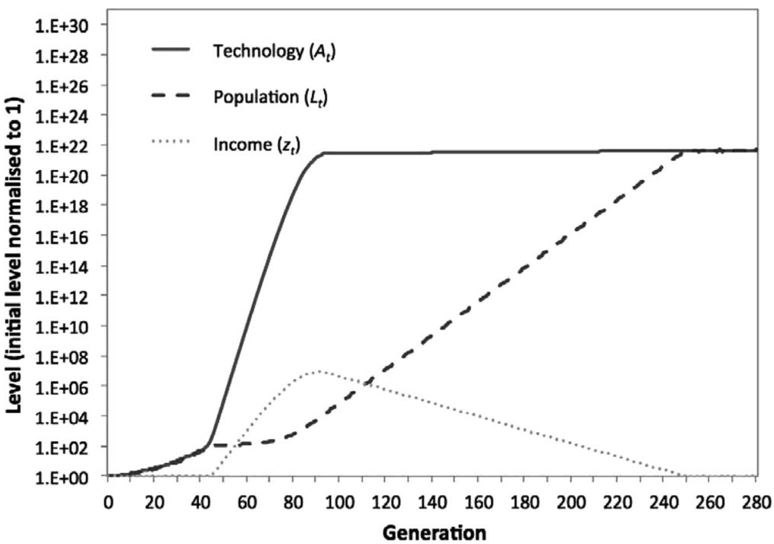

FIGURE 9. Level of technology, population, and income.

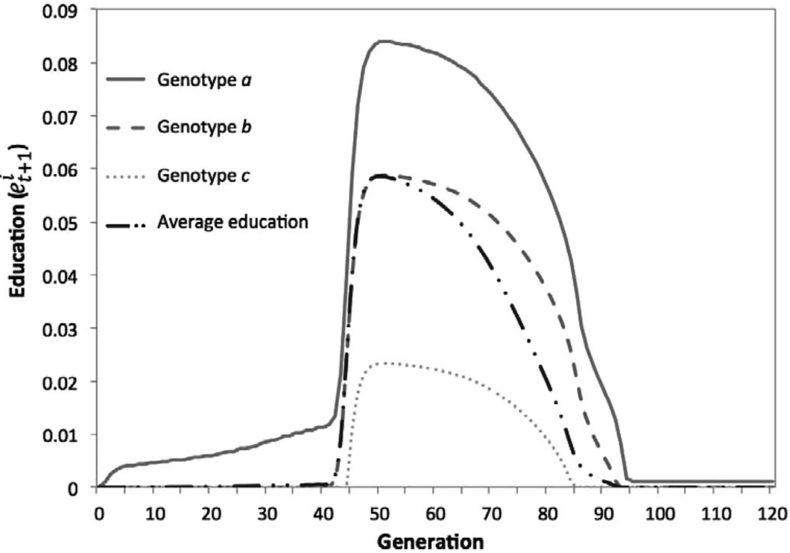

FIGURE 10. Education.

fitness as education increases potential income, and genotypes *b* and *c* have equal fitness as neither invests in education. Once the high-growth era commences, natural selection favors genotype *c* because the other two genotypes overeducate their children relative to the level of education that maximizes fitness. The return

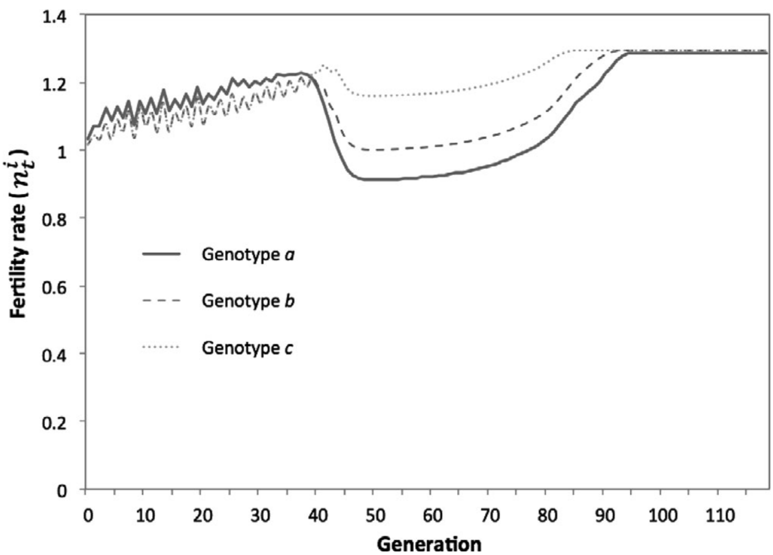

FIGURE 11. Fertility rate (five-generation moving average).

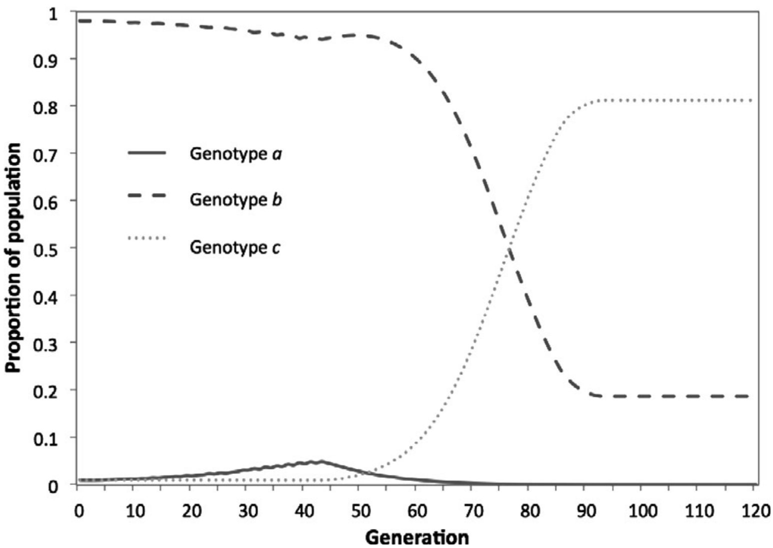

FIGURE 12. Proportion of population of each genotype.

to the Malthusian state is caused by a decline in per capita human capital, which is driven by the higher fertility of genotype *c* and its increasing prevalence in the population. Because genotype *c* invests little in education, the average education level of the population declines and technological progress stalls. After the return to Malthusian conditions, the bulk of the population is genotype *c*, with a small proportion of genotype *b* and genotype *a* almost driven to extinction.

After the return to Malthusian conditions, genotype *a* at first regains its fitness advantage and again starts to increase as a proportion of the population. It takes several hundred generations for genotype *a* to recover to meaningful numbers from its near extinction at the beginning of the second Malthusian state.[13] The renewed increase in the prevalence of genotype *a* again promotes technological progress, but it is not sufficient for another exit from Malthusian stagnation. The second Malthusian state is permanent because technological progress is matched by population growth. Thus, the situation is different from the initial Malthusian state with a small number of genotype *c* individuals. In the second Malthusian state, there is a large proportion of genotype *c* whose fertility absorbs any increase in income. After generation 565, a growth cycle repeats itself about every 25 generations without ever leading out of Malthusian stagnation. The cycles are generated by the interaction between genotype *a*, which drives technological progress, and genotype *c*, whose high fertility dilutes the average level of income in the population.

The timing of the return to Malthusian conditions is subject to the strength of the preferences of the strongly quantity-preferring type and their initial prevalence in the population. Given the other parameters used in the simulation, the economy

returns to a Malthusian state for any value of $\beta^c < 0.808$, with the economy returning to the Malthusian state more quickly as β^c is reduced. For example, income growth is negative by generation 140 for $\beta^c = 0.8$, by generation 95 for $\beta^c = 0.75$ (as graphed previously), and by generation 80 where the strongly quantity-preferring genotype does not invest in education at all (which is the case for any $\beta^c < 0.644$). However, for a value of $\beta^c = 0.809$, there is no return to a Malthusian state and per-person income grows at 1.5% per year in equilibrium, albeit slower than the 2.3% immediately following the transition phase. Thus, the return to Malthusian conditions occurs only if the preference of genotype c for quantity is sufficiently strong.

The finding of a possible return to a Malthusian era does not alter materially if the new genotype is introduced later in the simulation period. For example, the strongly quantity-preferring genotype may emerge in a population during the period of economic growth, which creates opportunities for global migration. In this situation, the return to Malthusian conditions would be delayed to the extent that the introduction of the new genotype was delayed. A simulation was conducted with a quantity-preferring genotype with a preference parameter $\beta^c = 0.75$ composing 1% of the population. If the new genotype is introduced after the transition phase at generation 50, income growth is negative by generation 100. A counteracting shock that introduces a strongly quality-preferring genotype into the population has, however, no long-lasting effect because the quality-preferring type will always have lower fertility in the modern growth era. The stronger the preference for quality of children, the lower the fitness of a genotype and the more quickly it will be eliminated from the population.

The return to Malthusian conditions may be prevented, however, by a scale effect of the form that features prominently in unified growth theory and the related Galor and Weil (2000) model. A scale effect would provide an additional source of technological progress, particularly following the rapid population growth at the time of the take-off into the modern growth era. The technological progress resulting from the scale effect would also provide an ongoing incentive for sufficiently quality-preferring agents to continue to invest in the human capital of their children, thereby providing a further foundation for economic growth.

10. CONCLUSION

The simulation of the Galor and Moav model produces a pattern of income and population growth that resembles the period of Malthusian stagnation before the Industrial Revolution and the take-off into a modern growth era. Although the simulation demonstrated that model outcomes are sensitive to variations in the preference for quality and the fixed time cost of childrearing, a range of parameters exist for which the core features of the Industrial Revolution can be achieved. In particular, the increase in income over approximately six generations and the rapid demographic transition in response to the changing quality–quantity trade-off faced by the population agents reflect what is observed in Western Europe.

The simulations of the extended model demonstrate that, given the absence of a scale effect, the economy can regress to a Malthusian state because of an increasing prevalence of a strongly quantity-preferring genotype. If the model includes three genotypes with a wider range of preferences between quantity and quality of children, economic growth is transitory if the third genotype is sufficiently quantity-preferring. The high fitness of the quantity-preferring genotype eventually returns the economy to Malthusian conditions and in the case of the scenario simulated, the second Malthusian state is permanent.

The simulation exercise highlights other considerations relevant to a biological evolution theory of the Industrial Revolution. There may exist some degree of phenotypic plasticity, which—in the current context—is the ability of an individual with a given set of genes to change its behavior in response to environmental conditions.[14] This might involve greater flexibility in the response to technological progress, which could allow quality-preferring genotypes to reduce their response to technological progress when overinvestment in education impairs their fertility. This flexibility would enhance the robustness of the modern-growth state by allowing quality-preferring genotypes to maintain a larger share of the total population. However, in the simulations presented in the preceding, a genotype that does not invest in education when income is above subsistence will always have a fitness advantage and drive the population back towards the Malthusian state.

NOTES

1. Alchian (1950) and Nelson and Winter (1982) applied evolutionary concepts to the theory of the firm and industrial organization.

2. Increasing technological progress and variation in heritable preferences underlies the trade-off in the Galor and Moav model, whereas a substitution effect due to higher wages drives the trade-off proposed by Becker et al. (1990)

3. The trigger for the take-off in the Galor and Weil (2000) model is increasing technological progress with increasing population, whereas the Galor and Moav (2002) model relies on investment in education by the quality-preferring types in the population.

4. Clark's proposal followed from work published by Clark and Hamilton (2006) on the reproductive success of the wealthy in England.

5. Genetic diversity was measured using expected heterozygosity, an index of the probability that two individuals, selected at random from the relevant population, are genetically different from one another.

6. The founder effect is the loss of genetic diversity that occurs when a small subset of a larger population establishes a new population.

7. Using $F(e,g) = \beta h_e - \frac{h(e,g)}{\tau+e} = 0$,

$$\frac{de}{dg} = -\frac{F_g}{F_e} = -\frac{\beta^i h_{eg}\left(\tau + e^i_{t+1}\right) - h_g}{\beta^i h_{ee}\left(\tau + e^i_{t+1}\right) - h_e + \frac{h}{\left(\tau + e^i_{t+1}\right)}}.$$

The educational response curve slopes upward if $-\beta^i h_{ee}(\tau + e^i_{t+1}) + h_e > h/(\tau + e^i_{t+1})$.

8. In relation to other species, human reproductive strategy of even the quantity-preferring type would be described as strongly K. There is considerable debate in the literature as to the appropriateness of applying r/K selection theory within the human species [for example, see Graves (2002)].

9. The parameter *a* could also be used for this purpose, but reducing *a* tends to increase perturbations and increase the instability of the model.

10. Haveman and Wolfe (1995) estimated expenditure on children as a proportion of GDP as approximately 0.15.

11. $g^* = e^{\left(\frac{0.023\times 20}{\alpha}\right)} - 1 = e^{\left(\frac{0.023\times 20}{0.4}\right)} - 1.$

12. We owe this interpretation to a comment by Oded Galor on an earlier draft of this paper.

13. No graphs are shown for the dynamics in the second Malthusian state.

14. The distinction between genotype and phenotype takes account of the observation that organisms with the same genetic code may look or act differently due to environmental conditions during their development.

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